PHYSICAL REVIEW E VOLUME 58, NUMBER 6 DECEMBER 1998

Diffusion with intrinsic trapping in two-dimensional incompressible stochastic velocity fields

M. Vlad,[1,2] F. Spineanu,[1,2] J. H. Misguich,[1] and R. Balescu[3]
[1]Association Euratom–Commissariat à l'Energie Atomique sur la Fusion, Département de Recherche sur la Fusion Controlée, Commissariat à l'Energie Atomique Cadarache, F-13108 Saint-Paul-lez-Durance, France
[2]National Institute for Laser, Plasma and Radiation Physics, P.O. Box MG-36, Magurele, Bucharest, Romania
[3]Association Euratom–Etat Belge sur la Fusion, Université Libre de Bruxelles, Campus Plaine Boulevard du Triomphe, 1050 Bruxelles, Belgium
(Received 13 July 1998)

A statistical approach that applies to the high Kubo number regimes for particle diffusion in stochastic velocity fields is presented. This two-dimensional model describes the partial trapping of the particles in the stochastic field. The results are close to the numerical simulations and also to the estimations based on percolation theory. [S1063-651X(98)05712-2]

PACS number(s): 47.10.+g, 05.40.+j, 05.60.+w, 52.35.Ra

I. INTRODUCTION

The motion in stochastic velocity fields describes a rather large class of physical processes such as particle and energy transport in plasmas or passive scalar advection in turbulent fluids. The analysis of such turbulent diffusion in continuous velocity fields relies on the general problem of relating the Lagrangian and the Eulerian statistical quantities. The latter are defined as statistical averages evaluated at fixed points in the laboratory frame while the corresponding Lagrangian quantities are determined at points following the motion of fluid elements. This is, in a sense, the fundamental problem of turbulence. Taylor [1] has shown that the diffusion coefficient is the time integral of the two-point Lagrangian correlation of the stochastic velocity. If this integral is finite, the mean square displacement of the particles is asymptotically diffusive (linear in time). More complex processes (subdiffusive or superdiffusive) can also appear when the integral is zero or divergent. Since this early work there have been rather few analytical approaches and results to this problem (see the reviews of Lumley [2] and McComb [3]). The domain of validity of various theories is determined by the value of the Kubo number K. The latter is defined as the ratio of the average distance covered by the particles during the correlation time of the stochastic velocity field to its correlation length. From a physical point of view, the Kubo number is a measure of the particle's capacity of exploring the space structure of the velocity field before the latter changes. In mathematical terms it is a parameter that determines the importance of the Lagrangian nonlinearity introduced by the space dependence of the velocity field. In the quasilinear regime $K<1$, the results are well established: The Lagrangian correlation is determined using the Corrsin approximation [3,4] and the resulting diffusion coefficient has the scaling $D_{ql}\sim K^2$. In the nonlinear case $K>1$, all theoretical models (which actually are explicitly or implicitly based on the Corrsin approximation) lead to a Bohm-like diffusion coefficient $D_B\sim K$ [5–9]. This is not a correct result since it does not vanish in the limit of frozen turbulence as it should. There is only one qualitative theoretical estimation by Isichenko [10,11], which is based on an analogy with the problem of percolation in stochastic landscapes and determines the scaling law of the diffusion coefficient at high Kubo numbers as $D_I\sim K^{0.7}$. Extended studies based on direct numerical simulations of the trajectories have also been performed [12–16]. They confirm the Isichenko scaling for some spectrum of the turbulence [12,13]. Moreover, they provide detailed information about statistical characteristics of the trajectories.

We present here a statistical approach to the test particle diffusion in a Gaussian stochastic velocity field that provides an analytical approximation for the Lagrangian correlation which is valid over the whole range between the quasilinear regime and the nonlinear one. Thus the time-dependent diffusion coefficient is obtained (not only its scaling with K like in Ref. [10]). The main ingredient of the model is the concept of *decorrelation trajectory* which determines the dynamics of the decorrelation process. Its validity is proved by several characteristics. The most important is that the diffusion coefficient has a K dependence close to the "percolation" estimate in the nonlinear regime. In the small K limit the quasilinear result is recovered. Also, the Lagrangian correlation of the potential is correctly reproduced by the model as well as the shape of the Lagrangian correlation of the velocity components determined in a direct numerical simulation. It was shown [17,18] that the physical reason for the subunitary exponents in the Kubo number scaling of the diffusion coefficients is the trapping of the particles in the structure of the random field. Our model describes this complicated trapping process.

The paper is organized as follows. The problem is introduced in Sec. II. Then, in Sec. III, we present a discussion about the Corrsin approximation and the results obtained in this framework. We show that the possibilities of improving this approximation in order to extend its application range to the high Kubo numbers regime are rather closed. The decorrelation trajectory method is described in Sec. IV, while Sec. V is devoted to the presentation of the results we have obtained and their physical interpretation. The conclusions are summarized in Sec. VI.

II. EULERIAN AND LAGRANGIAN CORRELATIONS

We consider a two-dimensional Langevin equation

$$\frac{d\mathbf{x}(t)}{dt}=\mathbf{v}(\mathbf{x}(t),t), \quad \mathbf{x}(0)=\mathbf{0}, \tag{1}$$

where the velocity field $\mathbf{v}(\mathbf{x},t)$ is a space- $(\mathbf{x})$ and time- (t) dependent continuous function in each realization. We are interested in incompressible velocity fields $\nabla\cdot\mathbf{v}(\mathbf{x},t)=0$, as appearing, e.g., in an electrostatic turbulence of a magnetically confined plasma, where $\mathbf{v}$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity of the guiding centers of the charged particles. A stream function (or a potential) is introduced that determines the two components of the velocity as $\mathbf{v}(\mathbf{x},t)=-\nabla\phi(\mathbf{x},t)\times\mathbf{e}_z$. The stream function $\phi(\mathbf{x},t)$ is a stochastic field considered to be Gaussian, stationary, and homogeneous. Since the velocity components are the derivatives of the stream function, they are Gaussian, stationary, and homogeneous as well. The Eulerian averages of the stream function and velocity are zero. The *two-point Eulerian correlation* (EC) *function* of the stream function is given. This is a measurable quantity defined as the statistical average of the stream function in two points. We chose the following model for the EC:

$$E(\mathbf{x},t)\equiv\langle\phi(\mathbf{x}_1,t_1)\phi(\mathbf{x}_1+\mathbf{x},t_1+t)\rangle=\beta^2\mathcal{E}(\mathbf{x})\exp\left(-\frac{|t|}{\tau_c}\right). \tag{2}$$

Due to the stationarity and homogeneity conditions, this average depends only on the distance $\mathbf{x}$ between the two points and on the time interval $|t|$. Angular brackets denote the statistical average over the realizations of the stochastic stream function field, β is the amplitude of the stream function fluctuations, and τ_c is their correlation time. $\mathcal{E}(\mathbf{x})$ is a function that decays from $\mathcal{E}(\mathbf{0})=1$ (where it has a maximum) to zero when $|\mathbf{x}|\to\infty$; its form is left unspecified at this stage. We consider an isotropic turbulence and thus the EC is a function of $|\mathbf{x}|$ only. In terms of these parameters, the Kubo number is

$$K=V\tau_c/\lambda, \quad V=\beta/\lambda, \tag{3}$$

where V measures the amplitude of the fluctuating velocity and λ is the average wavelength determined from the Fourier transform of $E(\mathbf{x},t)$, which is the spectrum of the stream function fluctuations. The two-point EC of the velocity components and of the stream function with the velocity are obtained from $E(\mathbf{x},t)$ by appropriate derivatives:

$$E_{xx}=-\frac{\partial^2}{\partial y^2}E, \quad E_{yy}=-\frac{\partial^2}{\partial x^2}E, \quad E_{xy}=\frac{\partial^2}{\partial x\,\partial y}E,$$
$$E_{x\phi}=-E_{\phi x}=\frac{\partial}{\partial y}E, \quad E_{y\phi}=-E_{\phi y}=-\frac{\partial}{\partial x}E, \tag{4}$$

where $E_{ij}(\mathbf{x},t)\equiv\langle v_i(\mathbf{0},0)v_j(\mathbf{x},t)\rangle$ and $E_{\phi i}\equiv\langle\phi(\mathbf{0},0)v_i(\mathbf{x},t)\rangle$.

In principle, the solution of this problem consists of finding the probability density for the displacements at any time t. This is a very difficult task that is usually reduced to finding the mean square displacement (MSD) $\langle\mathbf{x}^2(t)\rangle$ and the diffusion coefficient. Using the formal solution of Eq. (1),

$$x_i(t)=\int_0^t d\tau\, v_i(\mathbf{x}(\tau),\tau),$$

Taylor [1] has found a general expression for the MSD:

$$\langle x_i^2(t)\rangle=\int_0^t d\tau_1\int_0^t d\tau_2 L_{ii}(\tau_1,\tau_2), \tag{5}$$

where

$$L_{ii}(\tau_1,\tau_2)\equiv\langle v_i(\mathbf{x}(\tau_1),\tau_1)v_i(\mathbf{x}(\tau_2),\tau_2)\rangle \tag{6}$$

is the *two-point Lagrangian correlation* (LC) *function* of the velocity components. In the stationary and homogeneous case one can consider that the LC is a function of the time interval $\tau=|\tau_1-\tau_2|$ and Eq. (5) reduces to

$$\langle x_i^2(t)\rangle=2\int_0^t d\tau\, L_{ii}(\tau)(t-\tau). \tag{7}$$

The diffusion coefficient defined as $D_i(t)\equiv\frac{1}{2}(d/dt)\langle x_i^2(t)\rangle$ is

$$D_i(t)=\int_0^t d\tau\, L_{ii}(\tau). \tag{8}$$

Thus the Lagrangian correlation $L_{ii}(\tau)$ determines both the diffusion coefficient and the MSD of the trajectories. Consequently, the problem reduces to the determination of the Lagrangian correlation of the velocity components corresponding to the given Eulerian correlation of the stream function. We present an analytical approximation (the decorrelation path method) to solve this problem for any value of the Kubo number K. In order to give a better understanding of the physical significance of the method, we start with a short description of the Corrsin approximation and the results obtained in this frame.

III. THE CORRSIN APPROXIMATION

The Corrsin approximation [3,4] was extensively used in fluid and plasma physics in the past 30 years. It consists of two hypotheses: (i) The particle trajectories are statistically independent of the stochastic velocity field and (ii) the displacements have a Gaussian distribution.

In this framework, the LC of the velocity components is obtained as

$$L_{ij}(t)=\int d\mathbf{x}\, E_{ij}(\mathbf{x},t)P(\mathbf{x},t), \tag{9}$$

where $P(\mathbf{x},t)$ is the probability density for the displacements $\mathbf{x}$ in the time interval t, which, according to assumption (ii), is of Gaussian type:

$$P(\mathbf{x},t)\equiv\frac{1}{2\pi\langle x^2(t)\rangle}\exp\left(-\frac{\mathbf{x}^2}{2\langle x^2(t)\rangle}\right). \tag{10}$$

Since the mean square displacement $\langle x^2(t)\rangle$ is determined by the Lagrangian correlation [Eq.(7)], a closed set of equations is obtained. The LC is thus determined as the solution of Eqs. (9), (10), and (7).

One can easily see that in the quasilinear limit when $K \ll 1$, the MSD during the correlation time τ_c is much smaller than λ^2 and the resulting narrow probability density can be approximated in the integral in Eq. (9) by $\delta(\mathbf{x})$. The LC is then given by

$$L_{ij}(t) \cong E_{ij}(\mathbf{0},t), \quad K \ll 1, \tag{11}$$

which determines a diffusion coefficient

$$D_{ql} \cong V^2 \tau_c = (\lambda^2/\tau_c)K^2. \tag{12}$$

This is the well established quasilinear result. The Corrsin approximation is very good in the range $K<1$ and it can determine perturbative corrections of the diffusion coefficient (12).

At large K, Eq. (9) determines the narrowing of the LC (whose width decreases from τ_c to λ/V) and thus a slower dependence of the diffusion coefficient on the Kubo number is obtained. It was shown [5–9] that the scaling of the diffusion is of Bohm type:

$$D_B \sim V\lambda = (\lambda^2/\tau_c)K. \tag{13}$$

A fundamental criticism of this result is that in the limit of frozen turbulence ($\tau_c \to \infty$) the diffusion coefficient (13) does not vanish. In that case all trajectories wind around fixed closed contour lines of the stream function and the MSD cannot grow linearly in time so that the asymptotic diffusion coefficient has to be zero. The numerical simulations [12,13] confirm this idea showing that for some spectrum of the turbulence the scaling of the diffusion coefficient in K is

$$D_I \sim (\lambda^2/\tau_c)K^{0.7}, \tag{14}$$

as predicted by Isichenko [10]. It was shown [18] that in physical terms, the Bohm diffusion coefficient (13) (and consequently the Corrsin approximation) corresponds to neglecting the process of trajectory trapping in the structure of the stochastic stream function.

The origin of the Corrsin approximation (9) is the exact equation

$$L_{ij}(t) = \int d\mathbf{x}\, E_{ij}^c[\mathbf{x},t|\mathbf{x}(t)=\mathbf{x}]P(\mathbf{x},t), \tag{15}$$

where $E_{ij}^c[\mathbf{x},t|\mathbf{x}(t)=\mathbf{x}] \equiv \langle v_i(\mathbf{0},0)v_j(\mathbf{x},t)\rangle|_{\mathbf{x}(t)=\mathbf{x}}$ is the *conditional* correlation corresponding to the condition that the trajectory is at the point $\mathbf{x}$ at time t. This correlation has both Eulerian and Lagrangian properties: It is calculated at fixed points like the EC but depends on the trajectories like the LC. Using the first assumption of the Corrsin approximation the Lagrangian character of E_{ij}^c is neglected; thus E_{ij}^c is replaced by E_{ij}. We note that in the special case of Gaussian displacements it is possible to calculate the conditional correlation E_{ij}^c. Consequently, the first hypothesis can be eliminated and the LC can be determined under the restriction of the Gaussian hypothesis (ii) only. However, the resulting diffusion coefficient has, at large K, the same Bohm scaling (13) as obtained from the Corrsin approximation. The conclusion is that the displacements are not Gaussian at large K. This is confirmed by direct numerical simulations of particle trajectories [12,14,17], which show that the distribution of displacements $P(\mathbf{x},t)$ develops a peak around $\mathbf{x}=\mathbf{0}$ and large tails. This feature of $P(\mathbf{x},t)$ results from the process of particle trapping in the structure of the stochastic stream function: when the particles are moving near the maxima or minima of the stream function they wind for long time on almost closed paths of small size. Large displacements are performed only when they are in regions of small absolute values of the stream function.

The conclusion of this analysis is that the Gaussian assumption (ii) must be eliminated. However, for non-Gaussian displacements it is practically impossible to determine the conditional correlation E_{ij}^c (apart for some perturbative corrections that apply at $K \cong 1$ [19,20]). It follows that the possibilities for advancing beyond the Corrsin approximation (9) using the exact equation (15) are practically closed and that a completely different starting point should be found to study the diffusion in the high K nonlinear regime. The physical process that determines the reduction of the diffusion coefficient from Bohm to Isichenko scaling and also the non-Gaussian character of the displacements appears to be the trapping of the particles in the structure of the stochastic stream function. Thus the models must describe this complicated process explicitly.

IV. THE DECORRELATION PATH METHOD

The essential point of the present method is that it finds a set of deterministic trajectories that are determined by the EC of the stream function; the LC of the velocity is then approximated using the average velocity on these trajectories. The idea is to divide the space of realizations of the stochastic stream function into subensembles characterized by given values of the stream function and the velocity at the starting point of the trajectories:

$$\phi(\mathbf{0},0) = \phi^0, \quad \mathbf{v}(\mathbf{0},0) = \mathbf{v}^0. \tag{16}$$

The Eulerian correlation of the velocity components $E_{ij}(\mathbf{x},t)$ can be decomposed into a weighted sum of the Eulerian correlations of the velocity in each subensemble:

$$E_{ij}(\mathbf{x},t) = \int\int d\phi^0 d\mathbf{v}^0 P_1(\phi^0)P_1(\mathbf{v}^0)E_{ij}^s(\mathbf{x},t), \tag{17}$$

where $E_{ij}^s(\mathbf{x},t) \equiv \langle v_i(\mathbf{0},0)v_j(\mathbf{x},t)\rangle|_{\phi^0,\mathbf{v}^0}$ is the subensemble Eulerian correlation, i.e., it is an average conditioned by Eq. (16). $P_1(\phi^0)$ and $P_1(\mathbf{v}^0)$ are the Gaussian probability densities for the initial stream function and the initial velocity, respectively. The stream function and the velocity are statistically independent at the same point: Their correlation (4) is zero in $\mathbf{x}=\mathbf{0}$ where $\mathcal{E}(\mathbf{x})$ has a maximum. Consequently, the probability density for having the condition (16) is $P_1(\phi^0)P_1(\mathbf{v}^0)$. We note that Eq. (17) is an exact equation. The Eulerian correlation in the subensemble can be written as $E_{ij}^s(\mathbf{x},t) = v_i^0\langle v_j(\mathbf{x},t)\rangle|_{\phi^0,\mathbf{v}^0}$, where $\langle v_j(\mathbf{x},t)\rangle|_{\phi^0,\mathbf{v}^0}$ is the Eulerian average velocity in the subensemble (16). The latter is determined using the Gaussian conditional probability density for having the velocity $\mathbf{v}$ in the point $(\mathbf{x},t)$ when the condition (16) is imposed:

$$P(\mathbf{v},\mathbf{x},t|\phi^0,\mathbf{v}^0) \equiv \frac{\langle \boldsymbol{\delta}(\mathbf{v}-\mathbf{v}(\mathbf{x},t))\,\delta(\phi^0-\phi(\mathbf{0},0))\,\boldsymbol{\delta}(\mathbf{v}^0-\mathbf{v}(\mathbf{0},0))\rangle}{\langle \delta(\phi^0-\phi(\mathbf{0},0))\,\boldsymbol{\delta}(\mathbf{v}^0-\mathbf{v}(\mathbf{0},0))\rangle}.$$

Straightforward calculations lead to

$$\langle \mathbf{v}(\mathbf{x},t)\rangle|_{\phi^0,\mathbf{v}^0}=\mathbf{f}(\mathbf{x};\phi^0,\mathbf{v}^0)\exp\left(-\frac{t}{\tau_c}\right), \tag{18}$$

where

$$f_x(\mathbf{x};\phi^0,\mathbf{v}^0)=-v_x^0\frac{\partial^2\mathcal{E}(\mathbf{x})}{\partial y^2}+v_y^0\frac{\partial^2\mathcal{E}(\mathbf{x})}{\partial x\,\partial y}-\phi^0\frac{\partial\mathcal{E}(\mathbf{x})}{\partial y},$$

$$f_y(\mathbf{x};\phi^0,\mathbf{v}^0)=v_x^0\frac{\partial^2\mathcal{E}(\mathbf{x})}{\partial x\,\partial y}-v_y^0\frac{\partial^2\mathcal{E}(\mathbf{x})}{\partial x^2}+\phi^0\frac{\partial\mathcal{E}(\mathbf{x})}{\partial x}.$$

Equation (18) exhibits the space-time structure of the correlated zone. The average velocity in the subensemble (16) is $\mathbf{v}^0$ in $\mathbf{x}=\mathbf{0}$ and $t=0$ [because $\mathcal{E}(\mathbf{x})$ has a maximum there] and it decays progressively to zero as the time and/or the distance grows. Both time [through the factor $\exp(-t/\tau_c)$] and distance [through the factor $\mathbf{f}(\mathbf{x};\phi^0,\mathbf{v}^0)$] determine the decorrelation of the velocity.

In the quasilinear case $K\ll 1$, the decorrelation is mainly temporal (on the time scale τ_c) and the space factor in the Eulerian correlation reduces to $\mathbf{v}^0$. Equation (18) becomes $\langle \mathbf{v}(\mathbf{x},t)\rangle|_{\phi^0,\mathbf{v}^0}\cong\mathbf{v}^0\exp(-t/\tau_c)$ and the well known quasilinear result [Eqs.(11) and (12)] is obtained.

In the nonlinear case $K>1$, the space decorrelation is important. Our method consists in determining the dynamics of decorrelation represented by a set of deterministic trajectories and to approximate the Lagrangian correlation by using these trajectories.

The time variation of the stochastic stream function determines the time decay of the Eulerian correlations. This is a linear term in the sense that in the absence of space dependence of the velocity in Eq. (1), the problem is linear and the Lagrangian correlation is simply determined by the EC at $\mathbf{x}=\mathbf{0}$ [Eq. (11)]. The space depending factor $\mathbf{f}$ that appears in the subensemble average velocity (18) describes the structure of the correlated zone. We determine the dynamics induced by this structure by solving the equation

$$\frac{d\mathbf{X}(t)}{dt}=\mathbf{f}(\mathbf{X}(t);\phi^0,\mathbf{v}^0), \quad \mathbf{X}(0)=\mathbf{0}. \tag{19}$$

The solution of this equation $\mathbf{X}(t;\phi^0,\mathbf{v}^0)$, which will be called *the space decorrelation trajectory*, determines the typical evolution in the correlated zone and the way to leave it. We note that $\mathbf{X}(t;\phi^0,\mathbf{v}^0)$ is not an approximation of the average particle trajectory in the subensemble: Rather, it is a deterministic trajectory that represents the dynamics of the space decorrelation. Using for simplicity a reference frame with the x axis directed along $\mathbf{v}^0$ and dimensionless quantities $\overline{\mathbf{X}}=\mathbf{X}/\lambda$, $\overline{t}=t/\tau_c$, $u=|\mathbf{v}^0|/V$, and $p\equiv\phi^0/|\mathbf{v}^0|\lambda$, Eq. (19) becomes

$$\frac{d\overline{X}}{d\tau}=-\frac{\partial}{\partial\overline{Y}}\left(\frac{\partial}{\partial\overline{Y}}+p\right)\mathcal{E}(\overline{X},\overline{Y}),$$
$$\frac{d\overline{Y}}{d\tau}=\frac{\partial}{\partial\overline{X}}\left(\frac{\partial}{\partial\overline{Y}}+p\right)\mathcal{E}(\overline{X},\overline{Y}), \tag{20}$$

where a time variable $\tau\equiv Ku\overline{t}$ is introduced. This system has a Hamiltonian structure determined by the incompressibility of the stochastic velocity field in Eq. (1). The Hamiltonian

$$H(\overline{X},\overline{Y})\equiv\left(\frac{\partial}{\partial\overline{Y}}+p\right)\mathcal{E}(\overline{X},\overline{Y}) \tag{21}$$

is independent of the Kubo number K and depends only on the parameter p, which is essentially the initial stream function ϕ^0. Thus the solution of Eqs. (20) is a function of only two variables $\tau\equiv Ku\overline{t}$ and p: $\overline{\mathbf{X}}(\overline{t})\equiv\overline{\mathbf{X}}(\tau,p)$. Depending on the EC and on the parameters u,p, two types of trajectories can be obtained: (a) trajectories on which the velocity goes to zero (decorrelates from $\mathbf{v}^0$) and (b) closed periodic trajectories. Type (a) trajectories escape from the correlated zone, while type (b) trajectories are confined in it. Type (b) trajectories describe the trapping in the structure of the velocity field.

We introduce the average velocity observed along the decorrelation trajectory in each subensemble (16). Since this trajectory is deterministic, the latter is obtained by replacing $\mathbf{x}$ by $\mathbf{X}(Ku t,p)$ on the right-hand side of Eq. (18):

$$\langle \mathbf{v}(\overline{\mathbf{X}}(\tau,p),\overline{t})\rangle|_{\phi^0,\mathbf{v}^0}=\left(\frac{\beta}{\lambda}\right)u\frac{d\overline{\mathbf{X}}(\tau,p)}{d\tau}\exp(-\overline{t}). \tag{22}$$

The approximation on which our model is based consists in considering that the Lagrangian correlation of the velocity components is a weighted sum of the correlations observed *along the decorrelation trajectories* in each subensemble (16). Namely, starting from the Eulerian frame equation (17) and using the conditionally averaged velocities on the decorrelation trajectories (22), we approximate the Lagrangian correlation as

$$L_{ij}(\overline{t})\cong\int\int d\phi^0 d\mathbf{v}^0 P_1(\phi^0)P_1(\mathbf{v}^0)v_i^0\langle v_j(\overline{\mathbf{X}}(\overline{t}),\overline{t})\rangle|_{\phi^0,\mathbf{v}^0}. \tag{23}$$

The validity of this approximation will be proved *a posteriori* by the results obtained by our model for several quantities. After straightforward calculations consisting of the integration over the orientation of $\mathbf{v}^0$ and the change of variable $\phi^0\to p$, one obtains $L_{xy}(\overline{t})\cong 0$ and

$$L_{xx}(\overline{t})\cong L_{yy}(\overline{t})\cong\left(\frac{\beta}{\lambda}\right)^2 G(K\overline{t})\exp(-\overline{t}), \tag{24}$$

where

$$G(K\overline{t})=\frac{1}{\sqrt{2\pi}}\int\int_0^\infty dp\,du\,u^4\exp\left(-\frac{u^2(1+p^2)}{2}\right)\frac{dX(\tau,p)}{d\tau}. \tag{25}$$

$X(\tau,p)$ is the x component of the decorrelation trajectories determined from Eq. (20) and $\tau\equiv Ku\overline{t}$. We note that the Corrsin approximation also determines this symmetry of the LC $[L_{ij}(t)=\delta_{ij}L(t)]$.

The running diffusion coefficient is

$$D(\overline{t};K)=\frac{\lambda^2}{\tau_c}K\int_0^{\overline{t}}d\theta\,G(\theta)\exp\left(-\frac{\theta}{K}\right) \tag{26}$$

and the asymptotic diffusion coefficient can be written as

$$D(K)=\frac{\lambda^2}{\tau_c}\int_0^\infty d\theta\,F(\theta)\exp\left(-\frac{\theta}{K}\right), \tag{27}$$

where $F(\theta)=\int_0^\theta G(\tau)d\tau$. Equations (24) and (25), together with the equations for the space decorrelation trajectories (20), form a closed system of equations for determining the Lagrangian correlation of the velocity.

V. TESTS AND RESULTS

An important test of the methods of studying the diffusion in incompressible velocity fields consists in applying them to determine the LC of the stream function. As the velocity is always tangential to the contour lines of the stream function in the incompressible flow (1), the time variation of the Lagrangian stream function is determined only by the explicit time dependence $d\phi(\mathbf{x}(t),t)/dt=\partial\phi(\mathbf{x}(t),t)/\partial t$ and its Lagrangian correlation is $L_\phi(t)\equiv\langle\phi(\mathbf{0},0)\phi(\mathbf{x}(t),t)\rangle=\beta^2\exp(-t/\tau_c)$, independent of the space factor $\mathcal{E}(\mathbf{x})$ in the Eulerian correlations and of K. It is easily shown that the decorrelation path model reproduces this property. Using the conditional probability density for the stream function value ϕ at the point $(\mathbf{x},t)$ when the condition (16) is imposed, the average Eulerian stream function is determined as

$$\langle\phi(\mathbf{x},t)\rangle|_{\phi^0,\mathbf{v}^0}=\beta u\left(\frac{\partial}{\partial y}+p\right)\mathcal{E}(\mathbf{x})\exp\left(-\frac{t}{\tau_c}\right). \tag{28}$$

The average stream function on the decorrelation trajectory is obtained by replacing $\mathbf{x}$ by the space decorrelation trajectory in this equation. Since the Hamiltonian (21) is a constant of the motion $H(X(t),Y(t))=H(0,0)=p$ and noting that, by definition, $\beta up=\phi^0$, one obtains

$$\langle\phi(\mathbf{X}(t),t)\rangle|_{\phi^0,\mathbf{v}^0}=\phi^0\exp\left(-\frac{t}{\tau_c}\right).$$

The LC of the stream function is determined by using an approximation equivalent to Eq. (23):

$$L_\phi(t)\cong\int\int d\phi^0 d\mathbf{v}^0 P_1(\phi^0)P_1(\mathbf{v}^0)\phi^0\langle\phi(\mathbf{X}(t),t)\rangle|_{\phi^0,\mathbf{v}^0},$$

which gives $L_\phi(t)=\beta^2\exp(-t/\tau_c)$, as it should. We note that the Corrsin approximation reproduces this result only in the limit of small Kubo numbers. At high Kubo numbers this approximation determines a much narrower LC of the stream function that depends on $\mathcal{E}(\mathbf{x})$ and K (its width decreases when K increases).

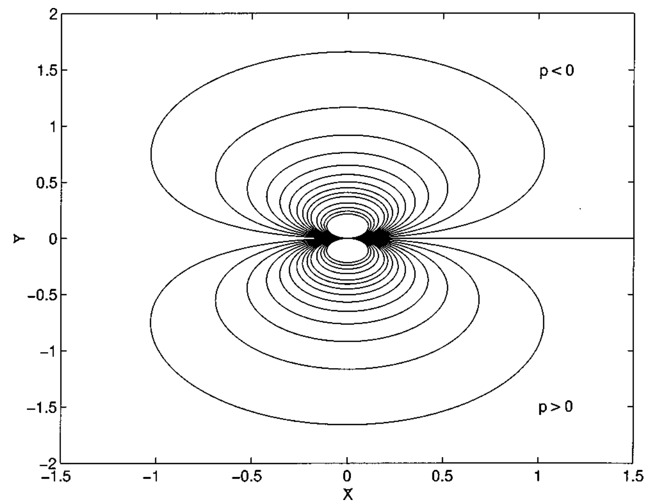

FIG. 1. Decorrelation paths for $p=0,\pm0.5,\pm1,\ldots$. The size of the curves decreases continuously with $|p|$.

In order to proceed, we have to specify the space dependence $\mathcal{E}(\mathbf{x})$ of the EC of the stream function (or, equivalently, the wave number spectrum). We model the EC of the stochastic stream function by

$$\mathcal{E}(\mathbf{x})=\frac{1}{(1+r^2/2n\lambda^2)^n}, \tag{29}$$

where $r^2=x^2+y^2$. This function ensures an amplitude of the velocity fluctuation $V=\sqrt{E_{ii}(\mathbf{0},0)}=\beta/\lambda$, as in Eqs. (3). We take $n=0.85$ to have a tail of the correlation similar to that considered in the numerical simulations [12,13]. A study of the dependence of the diffusion coefficient on n will be presented later. The Hamiltonian of the decorrelation paths (21) becomes

$$H(\mathbf{x})=\frac{1}{(1+r^2/2n)^n}\left(p-\frac{y}{1+r^2/2n}\right) \tag{30}$$

and using the invariance of $H(\mathbf{x})$ along the decorrelation trajectories, the system of equations (20) can be written as

$$\frac{dR}{d\tau}=\frac{1}{(1+R^2/2n)^{n+1}}\frac{X}{R},$$
$$Y=-p(1+R^2/2n)[(1+R^2/2n)^n-1], \tag{31}$$

where $R(\tau)=\sqrt{X^2(\tau)+Y^2(\tau)}$.

The decorrelation paths (31) are presented in Fig. 1 for several subensembles labeled by the values of p. All decorrelation paths are closed curves except the path for $p=0$, which is the straight line along $\mathbf{v}^0$. The size $R_{\max}$ of the paths grows continuously when the absolute value of p decreases. It can be approximated as $R_{\max}\cong2/|p|$ when $|p|\gg1$ and as $R_{\max}\cong(2n)^{(n+1)/(2n+1)}|p|^{-1/(2n+1)}$ when $|p|\ll1$. The decorrelation trajectories [resulting from the numerical integration of Eq. (20)] are periodic functions of $\tau\equiv Ku\overline{t}$ (except for p

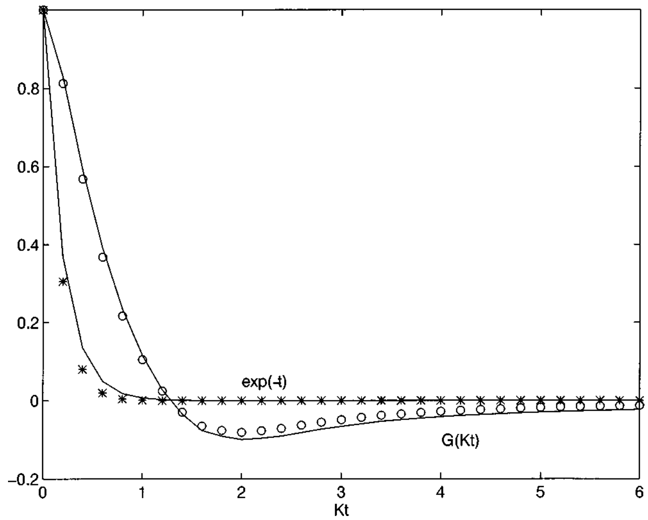

FIG. 2. LC for $K=0.2$ (stars) and 10 (circles) as a function of $K\overline{t}$, compared to the two factors in Eq. (24). At small K the LC is close to $\exp(-\overline{t})$, while at large K it has the shape of $G(K\overline{t})$.

$=0$). This shows that in the two-dimensional incompressible velocity fields the space decorrelation cannot be produced [practically all decorrelation trajectories are of type (b)] and a subdiffusive behavior is expected in the static case $\tau_c \to \infty$. This is a representation of the topology of the real flow that is characterized by a vortical motion (eddies).

The correlation of the Lagrangian velocity components is calculated according to Eqs. (24) and (25) using the decorrelation trajectories. Two time factors compete in determining the shape of the Lagrangian correlation (24): the exponential that accounts for the explicit time decorrelation and the function $G(Kt)$, which is determined by the Lagrangian nonlinearity. This function is calculated numerically and is presented in Fig. 2. The function $G(\theta)$ has a positive part (with a maximum at $\theta=0$) that decreases to zero at θ_0, followed by a negative minimum at θ_m and by a very long negative tail. The positive and negative parts have equal areas so that the integral of $G(\theta)$ is zero:

$$\int_0^\infty G(\theta)d\theta=0. \qquad (32)$$

This property can be deduced analytically. The function $G(\theta)$ is a representation of the space structure of the stochastic stream function: It is determined by the space correlation $\mathcal{E}(\mathbf{x})$. Actually, for the particular case of two-dimensisonal incompressible flows, the general shape of $G(\theta)$ is the same for all correlations (being the consequence of the incompressibility) and only the details (e.g., the values of θ_0 and θ_m) depend on $\mathcal{E}(\mathbf{x})$.

At small Kubo numbers the exponential factor prevails [$G(Kt)\cong 1$ in the range where the exponential is significantly different from zero]. The decorrelation is temporal and $L_{xx}(t)\cong(\beta/\lambda)^2\exp(-t/\tau_c)$. The nonlinear factor $G(Kt)$ becomes decisive at high Kubo numbers, where it provides a time variation faster than that of the exponential factor. The Lagrangian correlation $L_{xx}(t)$ at $K\gg 1$ has a shape similar to $G(Kt)$, but with the negative tail more attenuated because of the exponential factor. It has a zero at $t_0\cong\theta_0/K$ and a negative minimum at $t_m\cong\theta_m/K$. A direct comparison of the LC [Eq. (24)] with the results obtained in the numerical simulation [12,17] cannot be performed because the latter are obtained with a nonstationary (oscillatory) field energy. Moreover, the EC resulting from the wave number spectrum considered there is a very complicated function: It is not axisymmetric and has a large number of maxima and minima. However, as seen in Fig. 3, there is good qualitative agreement between the two results. Figure 3(a) presents the LC calculated for two values of the Kubo number [one in the nonlinear regime ($K=160$) and the other in the transition zone ($K=4$)] as a function of the scaled variable $K\overline{t}$. The numerical LC for the same values of K is shown in Fig. 3(b). One can see that in both cases the zero and the minimum of the LC scales as $1/K$, suggesting that the general expression of the LC [Eq. (24)] is correct. The oscillations observed in Fig. 3(b) on the tail of the LC are probably determined by the complicated shape of the corresponding EC, which has many maxima and minima.

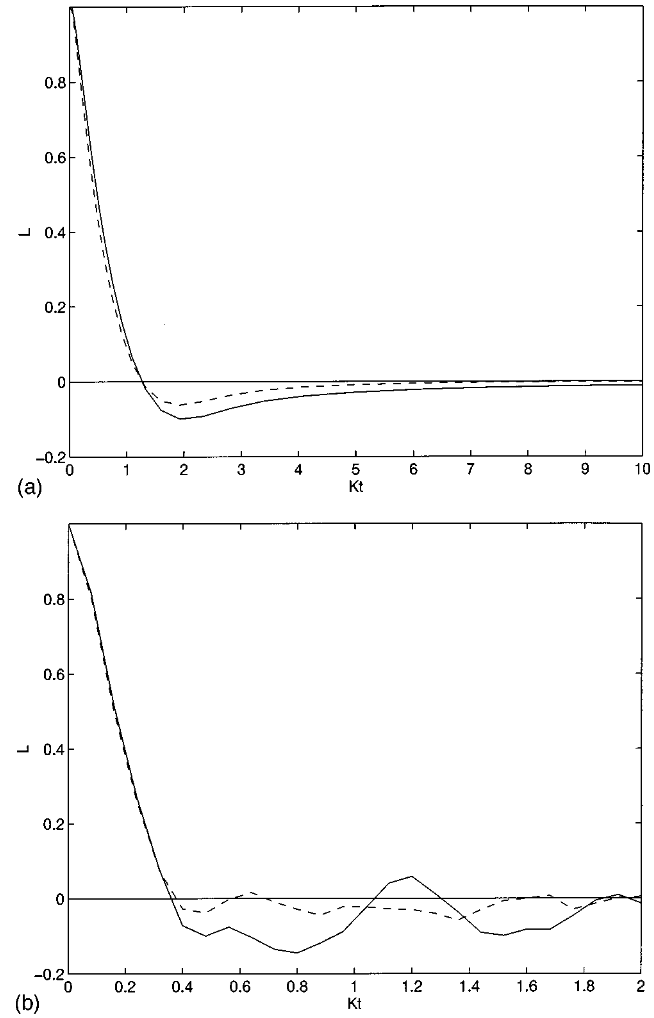

FIG. 3. LC (a) determined by the decorrelation path method and (b) calculated from the numerical simulations for $K=160$ (continuous lines) and 4 (dashed lines).

The large K numerical simulations of particle trajectories show that during their evolution the particles are temporarily trapped on almost closed, small size paths for durations long enough for performing a large number of rotations. Such

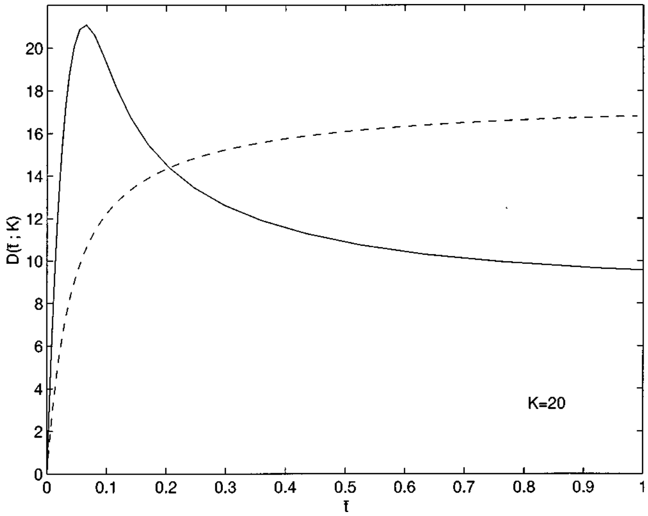

FIG. 4. Time-dependent running diffusion coefficient obtained with the decorrelation path method (continuous line) and with the Corrsin approximation (dashed line).

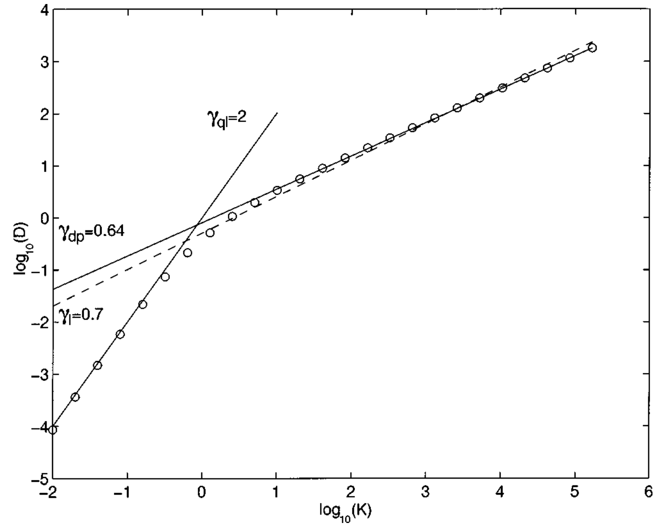

FIG. 5. Asymptotic diffusion coefficient as a function of the Kubo number compared to the quasilinear and percolation scaling.

trapping events appear around the extrema of the stream function, while long displacements are performed when the particles are moving at small absolute values of the stream function. Our model gives an image of this rather complicated trapping process that is actually contained in the Lagrangian correlation (24). The shape of the nonlinear factor $G(\theta)$ is determined by a selected contribution of the various paths (i.e., subensembles). The small paths with a large value of $|p|$ (i.e., of the stream function $|\phi^0|$) contribute only at the peak of $G(\theta)$ at $\theta=0$. At later times, since these trajectories perform a large number of rotations, their contributions cancel by an incoherent mixing in the integral over u. The negative tail of $G(\theta)$ results only from the contributions of the large paths corresponding to $|p|\ll 1$. When there is no time variation of the stochastic stream function ($\tau_c\to\infty$) the asymptotic diffusion coefficient is zero, $D\sim\int_0^\infty G(\theta)d\theta=0$, and the process is subdiffusive. A slow time variation ($\tau_c \gg 1$ or $K\gg 1$) produces the attenuation of the negative tail of $G(\theta)$, thus the elimination of the large paths contributes to the Lagrangian correlation, in other words, the decorrelation of those trajectories. A nonzero diffusion coefficient results from this release of the large size trajectories. Actually, the diffusion is produced only by the latter trajectories and not by the small ones whose contribution is not affected by the time decorrelation: Eddying regions, associated with the maxima of the stream function, continue to exist. When the time variation becomes fast ($\tau_c\ll 1$ or $K\ll 1$), all trajectories are decorrelated and the function G does not influence the diffusion coefficient. Thus the two factors evidenced in the Lagrangian correlation (24) have a clear physical interpretation. The nonlinear term G describes the trapping of particles near the extrema of the stream function while the linear factor $\exp(-t/\tau_c)$ accounts for the trajectory release. The Lagrangian correlation, and consequently the diffusion coefficient, results from the competition between trapping and release processes.

The trajectory trapping is also evident in the running diffusion coefficient. Figure 4 represents the function $D(t)$ calculated with the space decorrelation method, Eq. (26), and with the Corrsin approximation (9). One can see that important differences appear. The Corrsin diffusion coefficient grows continuously and saturates while the decorrelation path diffusion coefficient decays after reaching a maximum and finally saturates. This clearly shows that the diffusion is partly hindered due to trapping. The effect of trapping appears at times large enough so that the particles can explore the stochastic stream function.

The K dependence of the asymptotic diffusion coefficient (27) obtained for the Eulerian correlation (29) is presented in Fig. 5. After the small K quasilinear regime, a slower dependence on Kubo number is observed. The K dependence of the diffusion coefficient is weaker than in the Bohm scaling (confirming the presence of trajectory trapping in our model). The diffusion coefficient can be approximated as $D=0.81(\lambda^2/\tau_c)K^{0.64}$. As seen in Fig. 5, the results of our model are close to the percolation scaling [10] for a large range of the Kubo number from $K\cong 1$ to $K\cong 100\,000$. Considering the values of $K\lesssim 10^3$ as in the numerical simulation, the maximum relative difference between our results and their fitting with the percolation scaling is about 10%.

As these results were obtained for a particular choice of the EC of the stochastic stream function, a natural question has to be addressed: Does the diffusion coefficient and its scaling in K depend strongly on the shape of the EC or does it possess invariant features resulting from the qualitative analysis based on percolation [10]? In order to answer this question we have studied the dependence of the diffusion coefficient on the tail of the EC represented by the parameter n entering Eq. (29). The diffusion coefficients obtained for $n=0.85$, 1, 1.5, and 2 are presented in Fig. 6. One can see there a decrease of the diffusion coefficient with n, in the high K regime, but the dependence is weak. The exponents γ of the Kubo number scaling ($D\sim K^\gamma$) obtained for these values of n are $\gamma=0.64$, 0.62, 0.55, and 0.50. Thus the decorrelation trajectory method does not predict the invariance of the diffusion coefficient on the shape of the Eulerian correlation. However, we can say that the dependence of $D(K)$ on the EC of the stochastic stream function is not strong (in the range of K where the numerical calculations have been performed, namely, $K\lesssim 10^3$).

It should be noted that the decorrelation path method has several variants. This means that several sets of determinis-

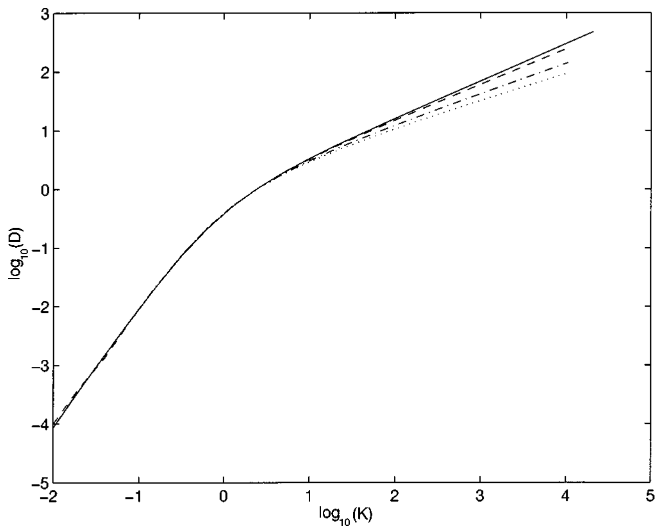

FIG. 6. Dependence of $D(K)$ on the parameter n: $n=0.85$ (continuous line), $n=1$ (dashed line), $n=1.5$ (dash-dotted line), and $n=2$ (dotted line).

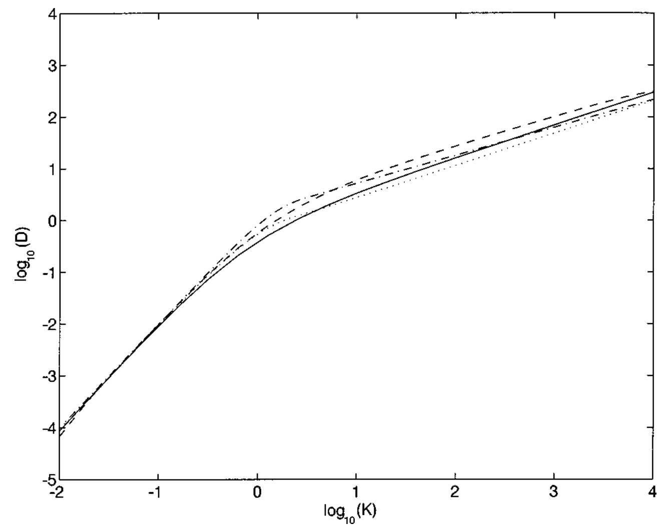

FIG. 7. Comparison of the variants of the decorrelation trajectory method: space decorrelation conditioned by $\phi^0,\mathbf{v}^0$ (continuous line); space-time decorrelation conditioned by $\phi^0,\mathbf{v}^0$ (dotted line); space decorrelation conditioned by ϕ^0 (dashed line); and space-time decorrelation conditioned by ϕ^0 (dash-dotted line).

tic, fictitious trajectories that describe the dynamics of the decorrelation process can be defined. Although they are rather different, they lead to compatible results, showing that the decorrelation path method is strong and robust. A possibility is to use, instead of the space decorrelation trajectories, the *space-time decorrelation trajectories*, which are defined starting from the subensemble average velocity (18), not from its space-dependent factor as in Eq. (19). Another possibility is to consider a different class of subensembles determined by the condition $\phi(\mathbf{0},0)=\phi^0$ instead of Eq. (16). This is rather natural since the statistics of the stream function determines completely the statistics of the velocity field. Both space and space-time decorrelation trajectories can be determined in these subensembles. A description of these variants of the decorrelation path method and a comparison of their results are presented in the Appendix. All these methods provide approximations for the LC and for the diffusion coefficient (see Fig. 7), but, according to the discussion presented in the Appendix, the space decorrelation trajectories in the subensembles with fixed stream function and velocity (Sec. IV) appear to give the better approximation.

We finally note that all the results presented here are generic for the two-dimensional, *incompressible* stochastic flows that appear to be characterized by a process of temporal trapping of the trajectories around the extrema of the stream function. However, the decorrelation trajectory method can also be used for other types of problems. In order to illustrate this statement we consider a *compressible* two-dimensional flow. It is described by the same Langevin equation (1), but a stream function cannot be introduced in this case. Thus the statistics of the velocity field should be specified: we consider a two-dimensional Gaussian velocity field that is stationary, homogeneous, and isotropic. The EC of the velocity components is modeled by

$$E_{xx}(\mathbf{x},t)=E_{yy}(\mathbf{x},t)=V^2\frac{1}{1+r^2/2\lambda^2}\exp\left(-\frac{t}{\tau_c}\right),\qquad E_{xy}(\mathbf{x},t)=0. \tag{33}$$

The space decorrelation trajectories are determined in subensembles conditioned by $\mathbf{v}(\mathbf{0},t)=\mathbf{v}^0$. All these trajectories are of type (b): They are straight lines along $\mathbf{v}^0$ and the velocity decays to zero as $t\to\infty$. Using the same approximation as in Sec. IV, one obtains the LC

$$L_{ii}(\overline{t})\cong\frac{1}{2}\left(\frac{\beta}{\lambda}\right)^2\exp(-\overline{t})\int_0^\infty dv\,v^3\exp\left(-\frac{v^2}{2}\right)\frac{1}{1+\frac{v^2}{2}\xi^2}, \tag{34}$$

where $\xi\equiv\xi(K\overline{t},v)$ is the dimensionless space decorrelation trajectory, which in this case is the solution of the algebraic equation $\xi+(v^2/6)\xi^3=K\overline{t}$. The LC is positive at all times, showing that the trapping on closed trajectories (the eddies) does not play a role in this case. Also the exponent γ of the Kubo number scaling is no longer subunitary. We have obtained here $\gamma=1.34$. Changing the EC space dependence (33) to $1/(1+r^n/\lambda^n)$, it can be shown that the exponent γ evolves from $\gamma_{ql}=2$ when $n\to 0$ to $\gamma_B=1$ when $n\to\infty$. The results are thus completely different from those obtained for the incompressible flow.

VI. CONCLUSIONS

We have presented a method, the decorrelation path method, by which to study particle diffusion in two-dimensional Gaussian incompressible stochastic velocity fields. This method is able to describe the complex process of diffusion and intrinsic trapping in the structure of the stochastic velocity field. We have introduced the concept of decorrelation trajectories that are determined from the Eulerian correlation of the stochastic field and describe the dynamics of the decorrelation process. The Lagrangian correlation of the velocity is approximated using the correlations observed along these fictitious, deterministic trajectories. We have obtained a general structure of the LC [Eqs. (24) and (25)], which shows that the effective diffusion results from a competition between trapping and release processes. In a frozen incompressible velocity field, the trapping is permanent and the particle evolution is subdiffusive. A weak time dependence of the stochastic velocity field ($K \gg 1$) produces the escape of a part of the trajectories, namely, of those corresponding to the small absolute values of the stream function, and the effective process becomes asymptotically diffusive. We have obtained a scaling of the diffusion coefficient with the Kubo number that is close to the numerical results and also to the percolation scaling. The decorrelation path method is a statistical approximation that yields results in agreement with the percolation scaling. Our tests show that this approximation is rather strong and robust and that it could be used in a large class of problems.

APPENDIX: OTHER APPROXIMATIONS

We present here other variants of the decorrelation trajectory method. They lead to rather different expressions for the diffusion coefficient, but the quantitative results are not very different. This shows that the decorrelation trajectory method is rather powerful and robust. A comparison of the results is presented in Fig. 7.

1. Space-time decorrelation

We have defined in Sec. IV the space decorrelation trajectories as deterministic trajectories that describe the dynamics of the space decorrelation in each subensemble (16). They are determined by the space dependence of the EC of the stream function field, i.e., by the wave number spectrum. It is also possible to determine another class of deterministic, fictitious trajectories, which will be called *space-time decorrelation trajectories*. They are solutions of

$$\frac{d\mathbf{X}'(t)}{dt} = \langle \mathbf{v}(\mathbf{X}'(t),t)\rangle|_{\phi^0,\mathbf{v}^0}. \quad \text{(A1)}$$

The subensemble average velocity on the right-hand side of this equation is given by Eq. (18). All these trajectories asymptotically saturate and the velocity along them goes to zero. They essentially determine the distance traveled in each subensemble before the time decorrelation takes place. In the time variable $\tau' \equiv Ku[1-\exp(-t/\tau_c)]$, Eq. (A1) is the same as Eq. (20) and the space-time decorrelation trajectories $\mathbf{X}'$ can be expressed in terms of the space decorrelation trajectories $\mathbf{X}$ as $\mathbf{X}'(t) = \mathbf{X}\{Ku[1-\exp(-t/\tau_c)],p\}$. The paths are the same, but for the trajectories $\mathbf{X}'(t)$ the period of winding around these paths increases continuously in time; eventually the trajectories stop. Approximating the LC of the velocity components by means of the subensemble average velocity along $\mathbf{X}'(t)$ as in Eq. (23), one obtains after straightforward calculations $L'_{ij}(t) = \delta_{ij}L'(t)$, where

$$L'(t) \cong \left(\frac{\beta}{\lambda}\right)^2 G\left\{K\left[1-\exp\left(-\frac{t}{\tau_c}\right)\right]\right\}\exp\left(-\frac{t}{\tau_c}\right) \quad \text{(A2)}$$

and the asymptotic diffusion coefficient

$$D'(K) = \frac{\lambda^2}{\tau_c} F(K)K, \quad \text{(A3)}$$

where F is the same function as in Eq. (27). Although these expressions are different from those obtained with the space decorrelation trajectories [Eqs. (24), (25), and (27)] the quantitative results are similar. The two diffusion coefficients [D' from Eq. (A3) and D from Eq. (27)] are the same at small K and have the same scaling in K in the high K regime. One can show analytically that $D' > D$ when K is of order 1 and $D > D'$ when $K \gg 1$, but the differences are not large (see Fig. 7). Thus both methods provide compatible approximations for the diffusion coefficient. However, a qualitative comparison of the LC (A2) and (24) with the LC determined from the direct numerical simulation of particle trajectories [Fig. 3(b)] shows that the results obtained with the space decorrelation trajectories are better. Equation (24) determines a $1/K$ dependence of the zero and of the minimum of the LC at intermediate and large values of K as observed in the numerical simulation (see Fig. 3). The space-time decorrelation result (A2) does not have this property.

2. Stream function conditioning

The stream function field $\phi(\mathbf{x},t)$ determines the two components of the velocity field and its statistics determines the statistics of the velocity. Consequently, the values of the stream function could be sufficient as a condition for determining the subensembles in the decorrelation path method. Indeed, a variant of the method can be constructed by eliminating the initial velocity in the condition (16). The conditionally averaged velocity is

$$\langle v_r(\mathbf{x},t)\rangle|_{\phi^0} = 0, \quad \langle v_\theta(\mathbf{x},t)\rangle|_{\phi^0} = \phi^0 \frac{d\mathcal{E}(r)}{dr}\exp\left(-\frac{t}{\tau_c}\right), \quad \text{(A4)}$$

where the polar coordinates (r,φ) for $\mathbf{x}$ are introduced. This determines particularly simple equations for the space decorrelation trajectories:

$$\frac{dR}{dt} = 0, \quad R\frac{d\varphi}{dt} = K\phi^0 \frac{d\mathcal{E}(R)}{dR}. \quad \text{(A5)}$$

They show that the decorrelation paths are concentric circles around the origin. The trajectory corresponding to the initial condition $\mathbf{X}(0) = \mathbf{0}$ is trivial [$\mathbf{X}(t) = \mathbf{0}$], so we need to consider nonzero initial conditions and integrate over the space. A measure has to be introduced in this integral: We assumed it to be the probability that a given point is on a contour line of $\phi(\mathbf{x},t)$ that has the linear size R. This is estimated in [10] as $P(R) \sim 1/R^2$ for large R. One obtains the LC

$$\mathcal{L}_{xx}(t) \cong \left(\frac{\beta}{\lambda}\right)^2 G_\phi(Kt)\exp\left(-\frac{t}{\tau_c}\right), \quad \text{(A6)}$$

where

$$G_\phi(\theta) \equiv \frac{1}{2}\int_0^\infty dR \frac{\mathcal{E}'^2}{R}\left(1 - \frac{\mathcal{E}'^2}{R^2}\theta^2\right)\exp\left(-\frac{\mathcal{E}'^2}{2R^2}\theta^2\right). \quad \text{(A7)}$$

$G_\phi(\theta)$, like $G(\theta)$ [see Eq.(25)], has a negative tail and a vanishing integral over θ. Also it shows that for large Kt only the large paths contribute in the LC.

Alternatively, we can determine the space-time decorrelation trajectories in the subensemble ϕ^0 (as in Sec. 1 of the Appendix) and the LC results as

$$\mathcal{L}'_{xx}(t) \cong \left(\frac{\beta}{\lambda}\right)^2 G_\phi \left\{ K \left[1 - \exp\left(- \frac{t}{\tau_c} \right) \right] \right\} \exp\left(- \frac{t}{\tau_c} \right). \tag{A8}$$

Figure 7 shows that both Eqs. (A7) and (A8) determine diffusion coefficients that are not very different from those obtained from the stream-function–velocity conditioning. Although this method has the advantage of very simple calculations, it needs external information (the measure for the space integration). The conditioning by both stream function and velocity eliminates this problem.

The conclusion of this discussion is that the idea of using a set of deterministic, fictitious trajectories determined from the EC of the fluctuating field provides a rather strong method for determining the LC and the diffusion coefficient. Several sets of such decorrelation trajectories determine compatible results. The space decorrelation trajectories in the subensembles with fixed stream function and velocity (Sec. IV) appear to give the better approximation.

[1] G. I. Taylor, Proc. London Math. Soc. **20**, 196 (1921).

[2] J. L. Lumley, in *Mécanique de la Turbulence,* International Conference of the CNRS, Marseille, 1962 (Centre National Recherche Scientifique, Paris, 1962).

[3] W. D. McComb, *The Physics of Fluid Turbulence* (Clarendon, Oxford, 1990).

[4] S. Corrsin, in *Atmospheric Diffusion and Air Pollution*, edited by F. N. Frenkiel and P. A. Sheppard (Academic, New York, 1959).

[5] T. H. Dupree, Phys. Fluids **10**, 1049 (1967).

[6] R. Balescu, *Transport Processes in Plasmas* (North-Holland, Amsterdam, 1988), Vol. 2, p. 770.

[7] J. B. Taylor and B. McNamara, Phys. Fluids **14**, 1492 (1971).

[8] R. H. Kraichman and D. Montgomery, Rep. Prog. Phys. **43**, 547 (1980).

[9] J. H. Misguich *et al.*, Plasma Phys. Controlled Fusion **29**, 825 (1987).

[10] M. B. Isichenko, Rev. Mod. Phys. **64**, 961 (1992).

[11] M. B. Isichenko, Plasma Phys. Controlled Fusion **33**, 809 (1991).

[12] J.-D. Reuss and J. H. Misguich, Phys. Rev. E **54**, 1857 (1996).

[13] J.-D. Reuss, M. Vlad, and J. H. Misguich, Phys. Lett. A **241**, 94 (1998).

[14] M. Ottaviani, Europhys. Lett. **20**, 111 (1992).

[15] M. De Leener, Phys. Rev. E **50**, 502 (1994).

[16] B. Weyssow and M. De Leener (unpublished).

[17] J. H. Misguich, J.-D. Reuss, M. Vlad, and F. Spineanu, Physicalia Mag. **20**, 103 (1998).

[18] M. Vlad, J.-D. Reuss, F. Spineanu, and J. H. Misguich, J. Plasma Phys. **59**, 707 (1998).

[19] J. Weinstock, Phys. Fluids **19**, 11 (1976).

[20] R. H. Kraichman, J. Fluid Mech. **81**, 385 (1977).